Journal of Applied Mechanics and Technical Physics, Vol. 39, No. 6, 1998

LINEAR PROBLEM OF A HYDROFOIL MOVING UNDER THE FREE SURFACE OF A FINITE-DEPTH FLUID

S. I. Gorlov

UDC 532.59

A method of solving the plane linear problem of a steady-state irrotational flow about a body under the free surface of a heavy fluid of finite depth is developed. The boundary-value problem is formulated for a complex perturbed velocity and is reduced to a singular integral equation relative to the intensity of a vortex layer that models the hydrofoil. The kernel of the equation is the exact solution of the corresponding boundary-value problem for a vortex of unit intensity. The equation is solved by the discrete-vortex method. The effect of the parameters of the problem on the hydrodynamic characteristics of the elliptical hydrofoil and the shape of the free surface are estimated numerically.

The problems of the motion of a body under the free surface of a bounded-from-below fluid have found many practical applications primarily concerning the design of ships moving in shallow water, and therefore many studies have focused on this problem. The first fundamental results were obtained by Tikhonov and Haskind [1, 2]. The success in the solution of these problems is connected with the development of numerical methods [3]. Bai [4] solved the problem of the motion of an elliptical cylinder and the Joukowski airfoil under the free surface of a heavy fluid of finite depth by the finite-element method. The panel method was applied by Giesing and Smith [5]. The shapes of the free surface of finite-depth and unbounded-from-below fluids were compared, and the wave effect on the distributed hydrodynamic loads was investigated. Using finite superelements, Mei and Chen [6] considered the problem of calculation of the wave drag and the lift force of a circular cylinder moving in a heavy fluid in the presence of a bottom. The hydrodynamic loads were found to undergo a discontinuity near the critical Froude number. For the solution of this problem, Taylor and Wu [7] employed the hybrid finite-element method. The flow of a heavy fluid with free surface about an elliptical cylinder and a hydrofoil at the angle of attack in the presence of the bottom was studied by Yeung and Bouger in [8], where the problem was solved using the method of hybrid boundary integral equations and the calculation results for the shape of the free surface and for the distributed and total hydrodynamic characteristics of the hydrofoil were given.

Despite the many studies in this field, a number of questions have not yet been clarified. In particular, there are scarce calculated data for assessing the effect of the generated waves on the hydrodynamic response of a body. The present work is devoted to the development of a numerical method of calculating the parameters of the problem in a wide range of variation and evaluation of the effect of the bottom on the total and distributed hydrodynamic characteristics of the hydrofoil.

We consider the linear problem of the uniform motion of an elliptical hydrofoil L under the free surface of a heavy fluid of finite depth. We assume that the fluid is ideal and incompressible. A coordinate system is introduced such that the x axis coincides with the unperturbed level of the free surface. The problem is solved in the plane of the complex variable $z = x + iy$. The acceleration of gravity is denoted by g, the density by ρ, the depth by H, the fluid velocity at an infinite distance in front of the hydrofoil by V_∞, the principal and

Omsk Division of the Sobolev Institute of Mathematics, Omsk 644099. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 39, No. 6, pp. 85–90, November–December, 1998. Original article submitted December 16, 1996; revision submitted March 31, 1997.

 0021-8944/98/3906-0892 $20.00

TABLE 1

$V_\infty/\sqrt{gh}$	$R_x/\rho a V_\infty^2$		$R_y/\rho a V_\infty^2$	
	Author	[12]	Author	[12]
0	0.00000	0.00000	0.22920	0.22977
0.2	0.00000	0.00000	0.36359	0.36426
0.4	0.00692	0.00674	0.40949	0.40374
0.6	0.65796	0.65971	0.87618	0.87869
0.8	1.02893	1.03159	0.14327	0.14388
1.0	0.64416	0.64808	−0.29479	−0.29539
1.2	0.34830	0.34918	−0.37174	−0.37257
1.4	0.18910	0.18957	−0.35120	−0.35202
1.6	0.10607	0.10633	−0.31563	−0.31637
1.8	0.06171	0.06186	−0.28421	−0.28488
2.0	0.03718	0.03726	−0.25978	−0.26041
∞	0.00000	0.00000	−0.16915	−0.16956

TABLE 2

x/h	$f(x)/h$	
	Author	[8]
−2.0	0.0634	0.0633
−1.0	0.1948	0.1945
−0.5	0.3244	0.3244
0	0.3932	0.3912
0.5	0.2415	0.2411
1.0	−0.0015	−0.0017
2.0	−0.3812	−0.3812
3.0	−0.6429	−0.6419
4.0	−0.8263	−0.8255

minor semi-axes of the ellipse by a and b, and the distance from the center of the ellipse to the free surface by h.

We introduce the complex velocity $\bar{V}(z)$ of the perturbed motion of the fluid. The function $\bar{V}(z)$ should be analytical in the domain D: $|x| < +\infty$ and $-H < y < 0$, except for the hydrofoil L, and should satisfy the following boundary conditions [9]:

(1) The consequence of the constant pressure and the zero normal velocity component on the free surface

$$\mathrm{Re}\left\{\frac{d\bar{V}(z)}{dz} i\nu \bar{V}(z)\right\} = 0, \quad \nu = \frac{g}{V_\infty^2} \quad \text{for} \quad z = x + i0; \tag{1}$$

(2) The zero normal velocity component at the bottom of the fluid

$$\mathrm{Im}\{\bar{V}(z)\} = 0 \quad \text{for} \quad z = x - iH; \tag{2}$$

(3) The no-flow at the hydrofoil points

$$\mathrm{Im}\{(V_\infty + \bar{V}(z))\, \mathrm{e}^{i\theta(z)}\} = 0 \quad \text{for} \quad z \in L, \tag{3}$$

where $\theta(z)$ is the angle between the tangent to L at the point z and the x axis;

(4) Damping of the perturbed velocities at an infinite distance in front of the hydrofoil

$$\lim_{x \to -\infty} \bar{V}(z) = 0. \tag{4}$$

We seek the perturbed complex velocity $\bar{V}(z)$ in the form

$$\bar{V}(z) = \int_L K(z,\zeta)\gamma(\zeta)\mathrm{e}^{-i\theta(\zeta)}\, d\zeta, \tag{5}$$

where $\gamma(\zeta)$ is the intensity of the attached vortex layer modeling the hydrofoil L ($\zeta = \xi + i\eta$). The expression $K(z,\zeta)$ obtained in [9] is the exact solution of the corresponding boundary-value problem (1), (2), and (4) for a vortex of unit intensity and has the form

$$K(z,\zeta) = \frac{1}{2\pi i}\frac{1}{z-\zeta} - \frac{1}{2\pi i}\frac{1}{z-\bar{\zeta}+2Hi} + \int_0^\infty \frac{\lambda+\nu}{\pi}\frac{\sinh(\lambda(H+\eta))\cos(\lambda(z-\xi+iH))}{\lambda\cosh\lambda H - \nu\sinh\lambda H}\mathrm{e}^{-\lambda H}\, d\lambda$$
$$+ \frac{\nu\sinh(\lambda_1(H+\eta))\sin(\lambda_1(z-\xi+iH))}{\nu H - \cosh^2\lambda_1 H}. \tag{6}$$

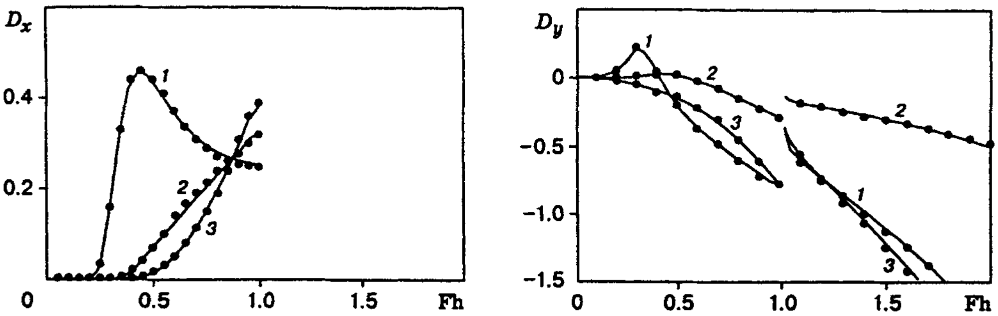

Fig. 1. Wave resistance $D_x = R_x/\rho g \pi a^2$ and the lift force $D_y = R_y/\rho g \pi a^2$ vs. the Froude number for $b/a = 1$, $H/a = 8$, and $h/a = 2$, 4, and 8 (curves 1–3): solid curves refer to the author's results, and the open points to the results of [6].

Here λ_1 is the positive root of the equation $\nu \tanh \lambda H = \lambda$, which exists for $\nu H > 1$.

Thus, the function $\bar{V}(z)$ satisfies conditions (1), (2), and (4), and the boundary-value problem (1)–(4) is reduced to the determination of $\gamma(\zeta)$ from the solution of the singular integral equation obtained by substituting (5) and (6) into (3). The integral equation is solved by the discrete-vortex method. We partition the hydrofoil L into elements $[z_j, z_{j+1}]$ of length Δ_j ($j = 1, \ldots, N$). At the beginning and end of each element, two vortices of intensity Γ_j and $-\Gamma_j$ are located. The total vortex intensity is zero, which ensures the irrotational flow about the ellipse. We choose the points z_{0k} ($k = 1, \ldots, N$) located in the center of the elements $[z_k, z_{k+1}]$ ($k = 1, \ldots, N$) and require satisfying the no-flow condition (3). According to formula (5), the function $\bar{V}(z)$ takes the following values at these points:

$$\bar{V}(z_{0k}) = \int_L K(z_{0k}, \zeta)\gamma(\zeta) e^{-i\theta(\zeta)} d\zeta. \tag{7}$$

We approximate the integral expression in (7) by the quadrature formula with discrete vortices [10]:

$$\int_L K(z_{0k}, \zeta)\gamma(\zeta) e^{-i\theta(\zeta)} d\zeta = \sum_{j=1}^{N} \Gamma_j (K(z_{0k}, z_j) - K(z_{0k}, z_{j+1})). \tag{8}$$

Substituting (7) and (8) into (3) for $z = z_{0k}$ ($k = 1, \ldots, N$), we obtain a system of N linear algebraic equations relative to the intensities of discrete vortices Γ_j ($j = 1, \ldots, N$).

To calculate the improper integral entering expression (6), an effective numerical method, which is based on the selection of the singularity from under the sign of the integral and the use of the Gauss and Laguerre quadratures [11], is developed.

After the intensities of the discrete vortices are found, with allowance for (7) and (8), at the hydrofoil points z_{0k} ($k = 1, \ldots, N$) we find the values of $\bar{V}(z)$ and the pressure p with the use of the Bernoulli integral:

$$p - p_\infty = -\frac{1}{2}\rho\Big(|V_\infty + \bar{V}(z)|^2 - V_\infty^2\Big),$$

where p_∞ is the pressure at an infinitely distant point.

To calculate the total hydrodynamic characteristics, we suggest a piecewise-constant pressure distribution along the hydrofoil L. The formulas for the wave drag R_x, the lift force R_y, and the moment M relative to the point $z_M = x_M + iy_M$ have the form

$$R_x - iR_y = i\int_L (p - p_\infty)\, e^{-i\theta} ds = i\sum_{k=1}^{N} (p_k - p_\infty)\, e^{-i\theta_k} \Delta_k,$$

$$M = -\int_L (p - p_\infty)[(\xi - x_M)\cos\theta + (\eta - y_M)\sin\theta]\, ds = -\sum_{k=1}^{N} (p_k - p_\infty)[(x_{0k} - x_M)\cos\theta_k + (y_{0k} - y_M)\sin\theta_k]\,\Delta_k,$$

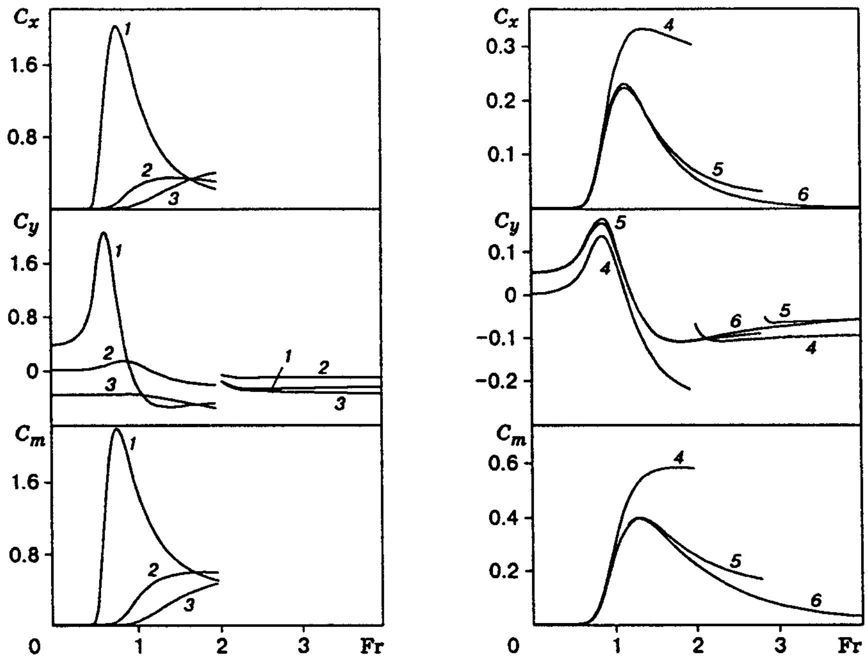

Fig. 2. Hydrodynamic characteristics vs. the Froude number for an elliptical hydrofoil ($b/a = 0.5$) for $H/a = 4$, $h/a = 1$, 2, and 3 (curves 1–3) and $h/a = 2$, $H/a = 4$, 8, and 20 (curves 4–6).

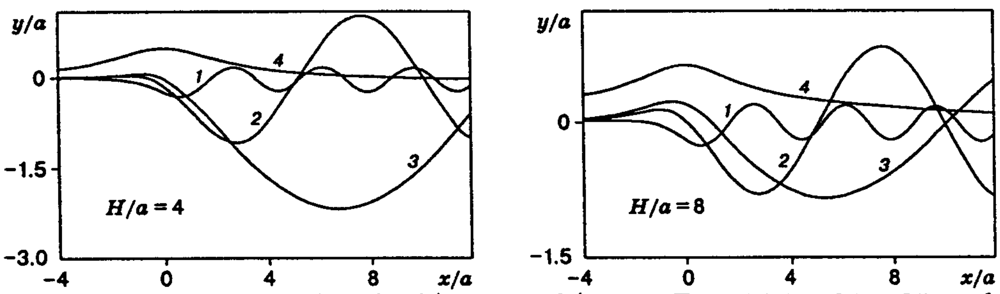

Fig. 3. Shape of the free surface for $b/a = 0.5$, $h/a = 2$, Fr = 0.75, 1.25, 1.75, and 3.25 (curves 1–4).

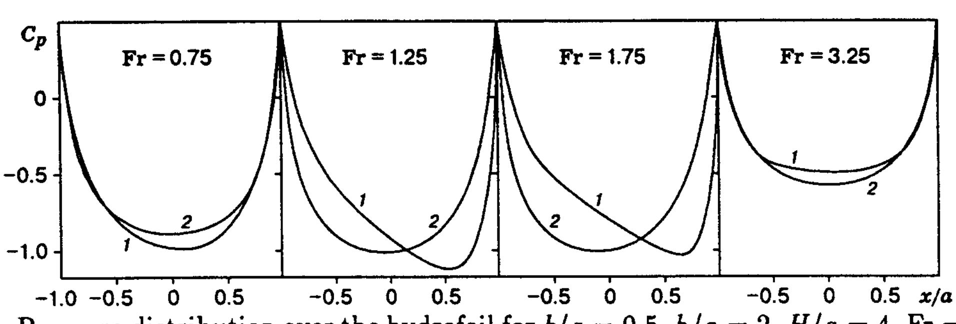

Fig. 4. Pressure distribution over the hydrofoil for $b/a = 0.5$, $h/a = 2$, $H/a = 4$, Fr = 0.75, 1.25, 1.75, and 3.25 (curve 1 refers to the upper side of the hydrofoil and curve 2 to the lower side).

where p_k is the pressure at the point z_{0k} and $\theta_k = \theta(z_{0k})$ $(k = 1, \ldots, N)$.

The shape of the free surface is set by the function $y = f(x)$, where $f(x) = -\mathrm{Re}\{\bar{V}(z)\}/\nu$ for $z = x+i0$. Here $\bar{V}(z)$ is found from (5) with allowance for the quadrature formula (8).

The basic parameters of the problem are as follows: b/a is the ratio of the semi-axis of the ellipse, h/a is the immersion of the ellipse, H/a is the depth of the fluid, and the Froude number is $\mathrm{Fr} = V_\infty/\sqrt{ga}$. Based on the developed algorithm, we performed a numerical experiment to evaluate the effect of these parameters on the wave-drag coefficients $C_x = 2R_x/\rho a V_\infty^2$, the lift force $C_y = 2R_y/\rho a V_\infty^2$, the moment relative to the center of the ellipse $C_m = 2M/\rho a^2 V_\infty^2$, and the pressure distribution over the hydrofoil and the channel bottom $C_p = (p - p_\infty)/\rho V_\infty^2$.

The calculational algorithm was tested by the known exact solution of the problem of an unbounded fluid flow about the ellipse. The number of partitions was taken to be equal to 60, and this gave a relative calculation error of less than 1% for $b/a > 0.1$.

The results were compared with the known solutions of the problem of a flow about a circular cylinder under the free surface of a heavy fluid of finite or infinite depth. Table 1 lists the calculated hydrodynamic characteristics of a circular cylinder for a bottomless fluid and the results obtained in [12]; it was assumed in the calculations that $h/a = 2$ and the depth $H/a = 20$, which corresponds to infinity. For a bottom-restricted fluid, Fig. 1 shows the wave drag and the lift force vs. the Froude number $\mathrm{Fh} = V_\infty/\sqrt{gH}$ and the results of [6]. The calculation data for the shape of the free surface and those obtained in [8] for $b/a = 1$, $H/a = 10$, $h/a = 2$, and $\mathrm{Fh} = 0.8$ are given in Table 2. The test calculations allow us to conclude that our results are in agreement with the known results.

The results of the systematic calculations, which were carried out to estimate the effect of hydrofoil-generated waves on its hydrodynamic characteristics, are given in Figs. 2–4.

Figure 2 illustrates the total hydrodynamic characteristics of the elliptical hydrofoil vs. the Froude number for various distances of the free surface and various depths of the fluid. Clearly, all the hydrodynamic characteristics undergo a discontinuity during the passage through the critical Froude number Fr_* (determined from the equation $\nu H = 1$) whose values for the three depths 4, 8, and 20 are 2, 2.828, and 4.472, respectively. As the fluid depth increases, the bottom effect becomes weaker and disappears for $H/a = 20$.

The dependence of the shape of the free surface on the Froude number is plotted in Fig. 3. As the critical Froude number is approached, one can observe a boundless increase in the amplitude and the wavelength. The calculation of the coefficient C_p at the channel bottom showed that, for $H/a = 4$, both the hydrofoil and the waves arising on the free surface contribute to the pressure distribution; the contribution of the waves becomes more pronounced as the critical Froude number is approached. For $H/a = 8$, this effect noticeably weakens. The waves arising on the free surface exert a strong effect on the pressure distribution on the upper side of the hydrofoil (Fig. 4).

Based on our numerical experiment, we can draw the following conclusions. The waves generated by a hydrofoil moving under the free surface of a heavy fluid of finite depth have a great effect on its total and distributed hydrodynamic characteristics. It is manifested particularly markedly in the vicinity of the critical Froude numbers, where the solution of the problem within the framework of the linear theory is not acceptable and requires an analysis in the nonlinear formulation. The bottom effect is very strong and disappears only for large values of the relative depth of the fluid.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 96-01-00093).

REFERENCES

1. A. I. Tikhonov, "Plane problem of the motion of a wing under the surface of a heavy fluid of finite depth," *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk*, No. 4, 57–78 (1940).
2. M. D. Haskind, "Translational motion of bodies under the free surface of a heavy fluid of finite depth," *Prikl. Mat. Mekh.*, **9**, 67–78 (1945).
3. R. W. Yeung, "Numerical methods in free-surface flows," *Ann. Rev. Fluid Mech.*, **14**, 395–442 (1982).

4. K. J. Bai, "A localized finite-element method for two-dimensional steady potential flows with a free surface," *J. Ship. Res.*, **22**, No. 4, 216–230 (1978).
5. J. P. Giesing and A. M. O. Smith, "Potential flow about two-dimensional hydrofoils," *J. Fluid Mech.*, **28**, No. 1, 113–129 (1967).
6. C. C. Mei and H. S. Chen, "A hybrid element method for steady linearized free-surface flows," *Int. J. Num. Meth. Eng.*, **10**, No. 5, 1153–1175 (1976).
7. Taylor R. Eatock and G. X. Wu, "Wave resistance and lift on cylinders by a coupled element technique," *Int. Shipbuild. Progr.,* **33**, No. 377, 2-9 (1986).
8. R. W. Yeung and Y. C. Bouger, "A hybrid integral-equation method for steady two-dimensional ship waves," *Int. J. Num. Meth. Eng.*, **14**, No 3, 317-336 (1979).
9. S. I. Gorlov, "Solution of the linear problems of the uniform motion of a vortex source in a multilayer fluid," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 3, 127–132 (1995).
10. S. M. Belotserkovskii and I. K. Lifanov, *Numerical Methods in Singular Integral Equations* [in Russian], Nauka, Moscow (1985).
11. S. I. Gorlov, "Plane problem of the motion of a body in a multilayer heavy fluid," Candidate's Dissertation in Phys. Math. Sci., Novosibirsk (1995).
12. T. I. Khabakhpasheva, "Plane problem of a uniform flow of a two-layer fluid about a circular cylinder," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 1, 91–97 (1996).